CALCULATION OF POSITION ERRORS dRMS, 2dRMS, CEP95 AND OF THE ERROR ELLIPSE FOR p=O.95

NAVGEN2001_08_std.MCD

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THE NAVGEN ALGORITHM

This Mathcad sheet contains a realisation of the NavGen algorithm to calculate the systematical and the random error of position fixes. The systematical error is calculated if the true position is known and used as reference position. Please read my paper: "A Standardized Algorithm for the Determination of Position Errors by the Example of GPS with and without Selective Availability" for further explanations.

1. Input of Data

Reading in of the data table containing the coordinates of the position fixes. The data file to be read can be produced from NMEA data using the GPS_NMEA_File_Parser available on the author's web site.

The data used here were collected in Bremen/Germany starting on 2001/09/16 at 12:55

Reading data table

$\mathrm{ORIGIN} \equiv 1$

W := PRNLESEN("L:\projekte\Navgen 20060917\NMEA_20060916_1300_GGA.prn")

$v := W^{\langle 1 \rangle}$

$N := \mathrm{letzte}(v)$ $N = 95523$ *Number of data sets in the table read in*

Sample rate and measurement time in seconds

Time for k samples $k := 10$

$$t_m := \begin{cases} \left(\mathrm{floor}\left(W_{k+1,1}\right) - \mathrm{floor}\left(W_{1,1}\right)\right) & \text{if } \left(\mathrm{floor}\left(W_{k+1,1}\right) - \mathrm{floor}\left(W_{1,1}\right)\right) \geq 0 \\ \mathrm{floor}\left(\max\left(W^{\langle 1\rangle}\right) - \mathrm{floor}\left(W_{1,1}\right)\right) + \mathrm{floor}\left(W_{k+1,1}\right) & \text{otherwise} \end{cases}$$

Measurement rate

$r_m := \frac{t_m}{10}$ $r_m = 1$ *second(s)*

Overall measurement time

$T_o := r_m \cdot N$ $T_o = 95523$ *seconds* $\frac{T_o}{3600} = 26.53$ *hours*

Definition of start number of data set and number of data sets to be processed

Start number $A_d := 1$

No of data sets to be processed $L_d := 86400$ *Insert here number of data sets to be processed or insert N !*

Loading of new table to be used for processing

$i := A_d .. \left(A_d + L_d - 1\right)$

$M_{i,1} := W_{i,1}$ $M_{i,2} := W_{i,2}$ $M_{i,3} := W_{i,3}$

$N := \mathrm{länge}\left(M^{\langle 1\rangle}\right)$ $N = 86400$ *Length of new table*

$t_M := \frac{N}{r_m \cdot 3600}$ $t_M = 24$ *Duration in h of data to be evaluated*

2. Reference Position

Here you can set the true position where the measurement has taken place, if it is known. In this case the systematical or calibration error will be calculated.
If the true position is not known, then the mean position will be determined from the data measured and this position will be used for the determination of the random error.

$wo := 1$ *This 'switch' must be set!*
wo:=1 true location is known; wo:=0 true location is not known and mean value of measured positions will be used as the reference position

$B_r := 53.07958761$
$L_r := 8.8720018$ *Insert true location, if known*

$mo := 1 - wo$ *If true location is unknown, the mean position will be used.*

3. Systematical Position Error (Calibration Error)

$B_m := \text{mittelwert}\left(M^{\langle 2 \rangle}\right)$ $B_m = 53.07958761$
$L_m := \text{mittelwert}\left(M^{\langle 3 \rangle}\right)$ $L_m = 8.8720018$ *Mean position*

$\max\left(M^{\langle 2 \rangle}\right) = 53.0796233$ $\min\left(M^{\langle 2 \rangle}\right) = 53.0795633$
$\max\left(M^{\langle 3 \rangle}\right) = 8.8720583$ $\min\left(M^{\langle 3 \rangle}\right) = 8.8719633$ *Extrema of fluctuations*

$B_r := wo \cdot B_r + mo \cdot B_m$ *Definition of reference position (either true or mean position)*
$L_r := wo \cdot L_r + mo \cdot L_m$ *setting check:* $wo = 1$ $mo = 0$

$B_d := B_m - B_r$ $B_d = 2.08402184 \times 10^{-11}$ *Deviation in Latitude*
$L_d := L_m - L_r$ $L_d = 1.86921412 \times 10^{-9}$ *Deviation in Longitude*

$B_k := 60 \cdot 1852 \cdot B_d$ $B_k = 0.00000232$ *Latitude deviation in metres*
$L_k := 60 \cdot 1852 \cdot L_d \cdot \cos\left(\frac{\pi}{180} \cdot B_r\right)$ $L_k = 0.00012477$ *Longitude deviation in metres*

Magnitude of systematical position error

$F_s := \sqrt{B_k^{\,2} + L_k^{\,2}}$ $F_s = 0$ *Distance from reference to measured position*

Bearing of systematical position error

$$\gamma := \text{atan}\left[\frac{(B_k)}{(L_k)}\right] \cdot \frac{180}{\pi}$$

$$\phi := \begin{cases} 90 - \gamma & \text{if } L_k > 0 \\ 270 - \gamma & \text{if } L_k < 0 \end{cases}$$

$B_k = 0.00000232$

$L_k = 0.00012477$

$\phi = 88.9$ *Bearing from reference to measured position*

4. Random Errors of Individual Positions

4.1 Deviations of individual measurements for the mean position

$M_{i,6} := 60 \cdot 1852 \cdot (M_{i,2} - B_m)$ *Deviations of y (N) coordinates from mean position*

$M_{i,7} := 60 \cdot 1852 \cdot (M_{i,3} - L_m) \cdot \cos\left(\frac{\pi}{180} \cdot B_m\right)$ *Deviations of x (E) coordinates from mean position*

$\max\left(M^{\langle 6 \rangle}\right) = 3.97$ $\min\left(M^{\langle 6 \rangle}\right) = -2.7$

$\max\left(M^{\langle 7 \rangle}\right) = 3.77$ $\min\left(M^{\langle 7 \rangle}\right) = -2.57$

Extrema of random deviations in metres

$\sigma_h := \text{stdev}\left(M^{\langle 6 \rangle}\right)$ $\sigma_h = 1.31$ *y (N)*

$\sigma_r := \text{stdev}\left(M^{\langle 7 \rangle}\right)$ $\sigma_r = 1.06$ *x (E)*

Standard deviations of the position coordinates in metres

4.2 Random error vectors of individual measurements (reference: mean position)

Magnitude

$$M_{i,10} := \sqrt{\left(M_{i,6}\right)^2 + \left(M_{i,7}\right)^2}$$

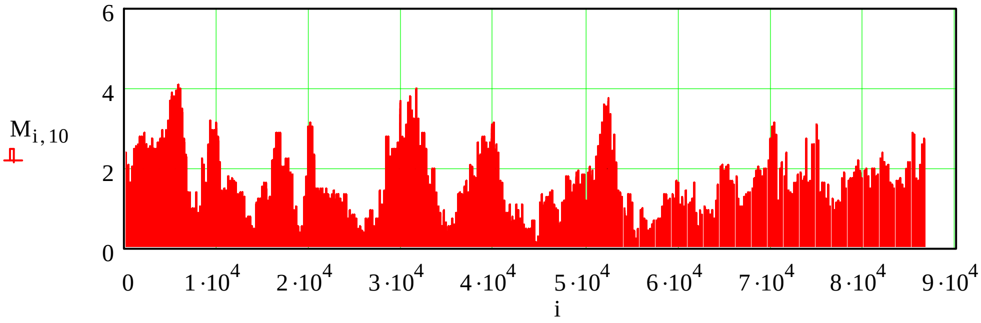
Bearing

$$\alpha_i := \text{atan}\left(\frac{M_{i,7}}{M_{i,6}}\right) \cdot \frac{180}{\pi}$$

$$M_{i,11} := \begin{cases} 90 - \alpha_i & \text{if } M_{i,7} > 0 \\ 270 - \alpha_i & \text{if } M_{i,7} < 0 \end{cases}$$

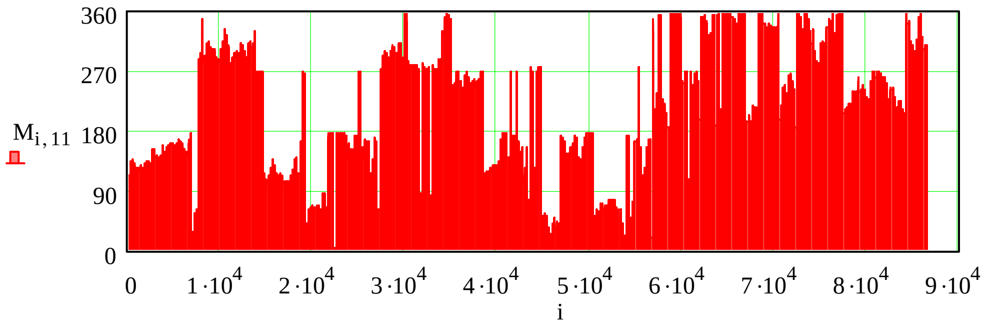
4.2.1 Plots of error vector magnitude and bearing

Time series of magnitudes



$\max\left(M^{\langle 10 \rangle}\right) = 4.09 \qquad \text{median}\left(M^{\langle 10 \rangle}\right) = 1.37 \qquad \text{stdev}\left(M^{\langle 10 \rangle}\right) = 0.79$

Time series of bearings



5. Error Circles of Varying Probabilities (dRMS, 2dRMS)

5.1 Excentricity of error ellipse

$$\text{std} := \begin{pmatrix} \sigma_h \\ \sigma_r \end{pmatrix}$$

$$k := \frac{\min(\text{std})}{\max(\text{std})} \qquad k = 0.81$$

5.2 dRMS position error

$$dRMS := \sqrt{\left(\sigma_h^{\ 2} + \sigma_r^{\ 2}\right)} \qquad dRMS = 1.68$$

Probability content of dRMS

$$P_{dRMS} := \frac{0.6300358 \cdot 20.672132 + 0.68259309 \cdot (k)^{(-5.1208746)}}{20.672132 + (k)^{(-5.1208746)}}$$

$P_{dRMS} = 0.637$ *(Harre approximation .2)*

5.3 2dRMS position error

$$2 \cdot dRMS = 3.36$$

Probability content of 2dRMS

$$P_{2dRMS} := 0.95435874 + 0.0017921523 \cdot k + 0.0895571 \cdot (k)^2 - 0.064296814 \cdot (k)^3$$

$P_{2dRMS} = 0.98$ *(Harre approximation .2)*

6. Error Circles of Fixed Probability (CEP95)

6.1 Decorrelation of measurement data

Correlation coefficient

$$\rho := \frac{\sum_{i=1}^{N} \left(M_{i,6} \cdot M_{i,7}\right)}{\sqrt{\sum_{i=1}^{N} \left(M_{i,6}\right)^2 \cdot \sum_{i=1}^{N} \left(M_{i,7}\right)^2}} \qquad \rho = -0.2475586591$$

Rotation angle for decorrelation

$$\Phi_r := \frac{1}{2} \cdot \mathrm{atan}\left(\frac{2 \cdot \rho \cdot \sigma_r \cdot \sigma_h}{\sigma_r^2 - \sigma_h^2}\right) \qquad \Phi_r = 0.4256412$$ *rad*

$$\Phi_d := \Phi_r \cdot \frac{180}{\pi} \qquad \Phi_d = 24.4$$ *deg*

6.2 Calculation of decorrelated position coordinates

$$\begin{pmatrix} M_{i,9} \\ M_{i,8} \end{pmatrix} := \begin{pmatrix} \cos(\Phi_r) & \sin(\Phi_r) \\ -\sin(\Phi_r) & \cos(\Phi_r) \end{pmatrix} \cdot \begin{pmatrix} M_{i,7} \\ M_{i,6} \end{pmatrix}$$

$$std := \begin{pmatrix} \mathrm{stdev}\left(M^{\langle 8 \rangle}\right) \\ \mathrm{stdev}\left(M^{\langle 9 \rangle}\right) \end{pmatrix} \qquad std = \begin{pmatrix} 1.37 \\ 0.98 \end{pmatrix}$$ *Standard deviations after decorrelation*

$$c := \frac{\min(std)}{\max(std)} \qquad c = 0.71640984$$ *Excentricity of error ellipse*

6.3 CEP95 - radius of error circle with p=0.95

$$k := 1.960787 + 0.004121 \cdot c + 0.114151 \cdot c^2 + 0.371707 \cdot c^3$$ *Factor for CEP95 (Harre approximation)*

$$CEP95 := k \cdot \max(std) \qquad CEP95 = 2.95$$

7. Visualisation of Error Contours

Coordinates of error circle CEP95

$$t := 1 .. 200 \cdot \pi$$

$$x_t := CEP95 \cdot \cos\left(\frac{t}{100}\right) + L_k$$

$$y_t := CEP95 \cdot \sin\left(\frac{t}{100}\right) + B_k$$

Coordinates of 95% error ellipse

$$\begin{pmatrix} u_t \\ v_t \end{pmatrix} := \begin{pmatrix} \cos(\Phi_r) & -\sin(\Phi_r) \\ \sin(\Phi_r) & \cos(\Phi_r) \end{pmatrix} \cdot \begin{pmatrix} 2.448 \cdot \sigma_r \cdot \cos\left(\frac{t}{100}\right) \\ 2.448 \cdot \sigma_h \cdot \sin\left(\frac{t}{100}\right) \end{pmatrix} + \begin{pmatrix} L_k \\ B_k \end{pmatrix}$$

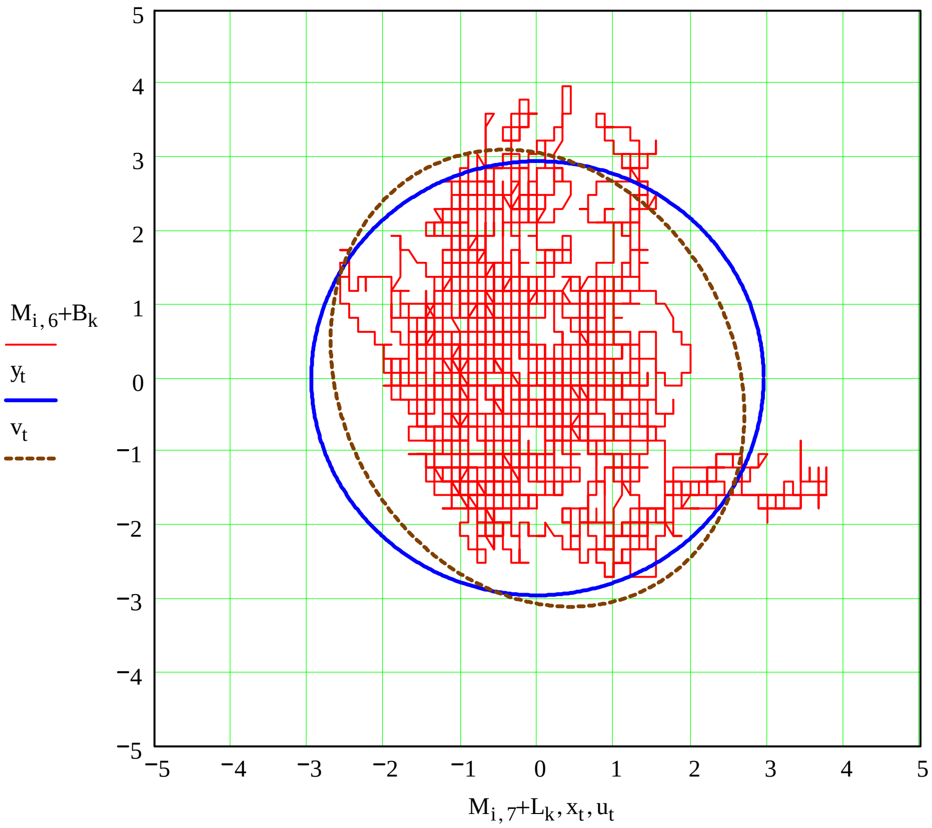
$2.448 \cdot \sigma_r = 2.58$

$2.448 \cdot \sigma_h = 3.2$

$\frac{\sigma_r}{\sigma_h} = 0.81$

$\Phi_r = 0.43$

$\Phi_d = 24.4$



Random errors of individual position measurements, error circle CEP95 and 95% error ellipse. Origin: reference position

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